

Quiz 02

September 17th, 2026

15 minutes | 5 points

Name _____

Instructions. Answer both questions. Show your work and briefly explain what each numerical answer means in context. Each part is worth 1 point. Use the space provided.

1. Calibrated sensor readings (Homework 01, Problem 2)

4 points

A sensor reading X has mean 20 and variance 4. The calibrated reading is

$$Y = \frac{3}{2}X - 2.$$

- (a) Compute $\mathbb{E}[Y]$.
- (b) Compute $\text{Var}(Y)$.
- (c) Now suppose $Y_1, \dots, Y_{16}$ are independent calibrated readings, each with the same distribution as Y .
Let

$$\bar{Y} = \frac{1}{16} \sum_{i=1}^{16} Y_i.$$

Compute $\mathbb{E}[\bar{Y}]$, $\text{Var}(\bar{Y})$, and the standard deviation of $\bar{Y}$.

- (d) Compare the standard deviation of one calibrated reading with that of the average. Briefly, explain what averaging accomplished.

Solution.

- (a) By linearity of expectation,

$$\mathbb{E}[Y] = \frac{3}{2}\mathbb{E}[X] - 2 = 28.$$

The calibrated reading has mean 28.

- (b) Adding a constant does not affect variance, while scaling by $3/2$ multiplies variance by $(3/2)^2$. Hence

$$\text{Var}(Y) = \left(\frac{3}{2}\right)^2 \text{Var}(X) = 9.$$

- (c) The average has expectation $\mathbb{E}[\bar{Y}] = 28$. Independence allows the variances to add, so

$$\text{Var}(\bar{Y}) = \frac{1}{16^2} \sum_{i=1}^{16} \text{Var}(Y_i) = \frac{9}{16}, \quad \sigma(\bar{Y}) = \sqrt{\frac{9}{16}} = \frac{3}{4}.$$

- (d) One calibrated reading has standard deviation 3, while the average has standard deviation $3/4$. Averaging 16 independent readings reduces the standard deviation by a factor of $\sqrt{16} = 4$ while preserving the mean.

2. Follow-up: choosing the sample size**1 points**

- (a) Instead of averaging 16 independent calibrated readings, suppose you average n independent calibrated readings. What is the smallest n for which the standard deviation of the average is at most 0.5?

Solution. The standard deviation of the average is $3/\sqrt{n}$. Thus

$$\frac{3}{\sqrt{n}} \leq 0.5 \iff \sqrt{n} \geq 6 \iff n \geq 36.$$

The smallest sample size is $n = 36$: averaging 36 independent calibrated readings gives standard deviation 0.5.